THEORETICAL AND PROBABILITY ANALYSIS OF FREQUENCY SELECTIVE CIRCUITS

G. Cherneva[1], A. Andonov[2]

[1] *Department of Electrical Engineering Higher School of Transport*
Geo Milev 158, 1584 Sofia, tel.: +359 896 718732, mail: cherneva@vtu.bg

[2] *Department of Communications and Safety Equipment and Systems, Higher School of Transport*
Geo Milev 158, 1584 Sofia, tel.: +359 899 889903, mail: andonov@vtu.bg

Summary The deviations of the circuit characteristics from their nominal values are random quantities and appear as a result of different destabilizing factors. With designing frequency selective circuits, two mutually connected problems are solved. The one is a study on the possible variations of circuit characteristics with given probability indices for the instability of the characteristics of their elements. The other is to synthesize the circuit and to determine the nominal parameters of its elements considering the requirements for stability of its characteristics. Both problems are solved by the methods of the theory of circuit sensitivity. The paper presents a new approach to theoretical probability analysis of the characteristics of frequency selective circuits depending on the relative changes of their parameters.

1. INTRODUCTION

With designing frequency selective circuits, it is necessary to meet severe requirements put not only to their frequency characteristics, but also to the admissible deviations from the nominal values, i.e. stability of those characteristics. As a result of the random variations of the circuit parameters causing those deviations they also are of a random nature. Each random quantity is determined by the law of distribution of probabilities [2] or its numerical characteristics – the moments of the first and second order [2]. With a normal Law of distribution of parameter variation probabilities, the law of distribution of the circuit schematic function is also normal. If the deviations of parameters do not subject to the normal law of distribution and are very small, then as a result of a theorem of the theory of probabilities [1], it can be assumed that the summed law of distribution of circuit function is close to the normal one. Hence, it is sufficient to determine the mathematical expectation and dispersion of the circuit schematic function. They are determined on the base of the theory of circuit sensitivity [3].

2. DETERMINING THE PROBABILITY CHARACTERISTICS FROM THEIR NOMINAL VALUES

The probability characteristics (moments) of the deviation from the nominal values of the schematic function of the kind [1,5]:

$$F(x, \vec{y}) = F\{x, y_1,....y_n\}, \tag{1}$$

where x is an independent variable: frequency, time, complex variable p,

$\vec{y} = \{y_1,....y_n\}$ is the vector of the parameters of circuit elements, are defined on the base of the method of moments [5].

Let Δy_i , $i = 1...n$, are the variations of the circuit parameters appearing as a result of different destabilizing factors. Then the relative change of the circuit function towards its nominal value can be presented in the order of T-series [2] in the kind of:

$$\frac{\Delta F}{F} \approx \sum_{i=1}^{n} S_i(x)\frac{\Delta y_i}{y_i} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} S_{ij}(x)\frac{\Delta y_i \Delta y_j}{y_i y_j}, \tag{2}$$

where

$$S_i = \frac{\partial F}{\partial y_i}\frac{y_i}{F}, S_{ij} = \frac{\partial^2 F}{\partial y_i \partial y_j}\frac{y_i y_j}{F} \tag{3}$$

are the circuit sensitivity functions of the first and second order related to the nominal parameters y_i.

Let deviations Δy_i are independent random values with a normal law of distribution of probabilities, with zero mathematical expectation $m\left[\frac{\Delta y_i}{y_i}\right] = 0$ and dispersion $D\left[\frac{\Delta y_i}{y_i}\right] = \sigma_i^2$. Then the probability characteristics of the schematic function variations are expressed from (2) in the kind of:

$$m\left[\frac{\Delta F}{F}\right] \cong \frac{1}{2}\sum_{i=1}^{n} S_{ii}\sigma_i^2 , \tag{4}$$

$$D\left[\frac{\Delta F}{F}\right] \cong \sum_{i=1}^{n} S_i^2\sigma_i^2 + \frac{1}{2}\sum_{i=1}^{n} S_{ii}^2\sigma_i^4 + \sum_{i=1}^{n}\sum_{j=1}^{n} S_{ij}^2\sigma_i^2\sigma_j^2 \tag{5}$$

With small and statistically independent relative deviations $\frac{\Delta y_i}{y_i}$, with mathematical expectations m_i and dispersions σ_i^2, it can be assumed [5] that the probability characteristics of the law of distribution of schematic function variations are:

$$m\left[\frac{\Delta F}{F}\right] \cong \sum_{i=1}^{n} S_i m_i , \tag{6}$$

$$D\left[\frac{\Delta F}{F}\right] \cong \sum_{i=1}^{n} S_i^2\sigma_i^2 . \tag{7}$$

The relative variations of the parameters $\frac{\Delta y_i}{y_i}$ that appear as a result of different external factors can be presented in the kind of [5] :

$$\frac{\Delta y_i}{y_i} = \alpha_i \Delta\theta \ , \quad i = 1...n \ , \tag{8}$$

where $\Delta\theta = \theta - \theta_0$ is the deviation of a given operational condition (e.g. temperature) from its normal value, θ_0, α_i is the factor of influence (e.g. temperature coefficient) of the element with a parameter y_i , n – number of elements.

If the factors of influence are random values with a with a normal law of distribution, mathematical expectation α_{i0} and dispersion d_{i0}^2, the probability characteristics of the variations of parameters can be presented in the kind of:

$$m\left[\frac{\Delta y_i}{y_i}\right] = \alpha_{i0} \Delta\theta , \tag{9}$$

$$D\left[\frac{\Delta y_i}{y_i}\right] = d_{i0}^2 \Delta\theta^2 \ . \tag{10}$$

In case that $\Delta\theta \in [-\Delta\theta_m , \Delta\theta_m]$, it can be written that

$$m\left[\frac{\Delta y_i}{y_i}\right] = \lambda m_i \quad , \tag{11}$$

$$D\left[\frac{\Delta y_i}{y_i}\right] = \lambda^2 \sigma_i^2 \quad , \tag{12}$$

where $m_i = \alpha_{i0}\Delta\theta$ and $\sigma_i^2 = d_{i0}^2 \Delta\theta^2$ are the boundary values of the mathematical expectation and the dispertion of its deviation Δy_i, a $\lambda \in [-1,1]$.

On the base of dependencies (6), (7), (11) and (12) for the probability characteristics of the circuit schematic function variations it is obtained that:

$$m\left[\frac{\Delta F}{F}\right] \cong \lambda \sum_{i=1}^{n} S_i m_i \quad , \tag{13}$$

$$D\left[\frac{\Delta F}{F}\right] \cong \lambda^2 \sum_{i=1}^{n} S_i^2 \sigma_i^2 \ . \tag{14}$$

The dependencies obtained (13) and (14) give a possibility to solve the problems of analysis and synthesis of the frequency selective circuits with considering the variations of their parameters.

The worked-out dependencies of the probability characteristics of the schematic function variations have been applied to LC circuit. The reactive elements are under relatively equal conditions related to the influence of environment. It is considered [1] that the temperature coefficients, the coefficients of aging and others influencing on the deviations of the elements factors are approximately one and the same for the elements of one kind. Because of that the probability characteristics of the deviation of their parameters are the same. In that sense, let set:

$$m_i = m_L \ , \ \sigma_i = \sigma_L \ , \ i = 1,...n_L \ . \tag{15}$$

for all inductivities of n_L number in the LC circuit in expressions (13) and (14). Analogously:

$$m_i = m_C \ , \ \sigma_i = \sigma_C \ , \ i = 1,...n_C \ . \tag{16}$$

is fulfiled for n_C nuber of capacities.

Then the probability characteristics of the schematic function variations are as follows:

$$m\left[\frac{\Delta F}{F}\right] \cong \lambda \left[m_L \sum_{i=1}^{n_L} S_{Li} + m_C \sum_{i=1}^{n_C} S_{Ci} \right], \tag{17}$$

$$D\left[\frac{\Delta F}{F}\right] \cong \lambda^2 \left[\sigma_L^2 \sum_{i=1}^{n_L} S_{Li}^2 + \sigma_C^2 \sum_{i=1}^{n_C} S_{Ci}^2 \right]. \tag{18}$$

It is seen that the sums of the sensitivity functions by elements of one and the same kind of the reactive chain characterize the average deviation, and the sums of their squares characterize the dispersion of the deviation with each value of the independent variable *x*.

3. EVALUATION OF ACCURACY WITH DETERMINING PROBABILITY CHARACTERISTICS

The worked-out equations (17) and (18) can be used for analysis of the stability of frequency selective circuits. However, these dependencies are approximate as they have been obtained as a result of Taylor's limitation to the second member in equation (2). The authenticity of the analysis depends on the size of the variations of parameters Δy_i. The absolute deviation of the schematic function can be expressed from equation (2) in the kind of:

$$\Delta F \approx \frac{\Delta y_i}{y_i} \frac{\partial F}{\partial y_i} y_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial F}{\partial y_i \partial y_j} y_i y_j \frac{\Delta y_i \Delta y_j}{y_i y_j} \tag{19}$$

On the base of the dependency (19), the approximate kind $F_1(\omega)$ of the chain frequency characteristics with variations of its parameters $\frac{\Delta y}{y} = \frac{\Delta L}{L} = \frac{\Delta C}{C}$ can be written as follows:

$$F_1(\omega) \approx F(\omega) + \frac{\Delta y}{y} F'(\omega).\omega + \frac{1}{2}\left(\frac{\Delta y}{y}\right)^2 F''(\omega).\omega^2 \ . \tag{20}$$

where $F'(\omega)$ and $F''(\omega)$ are its derivatives by frequency from then first and second order.

On the other side, the frequency characteristic variations can be determined as [3]:

$$\Delta F(\omega) = \left(1 + \frac{\Delta y}{y_0}\right) F(\omega) , \tag{21}$$

respectively

$$F_2(\omega) = F(\omega) + \left(1 + \frac{\Delta y}{y_0}\right) F(\omega) . \qquad (22)$$

The relative value with determining the variations of the frequency characteristic can be defined as:

$$\delta\% = \frac{F_2(\omega) - F_1(\omega)}{F_2(\omega)} .100\% .$$

4. CONCLUSION

The approach suggested for probability analysis of the frequency characteristics is based on the summed relative sensitivities of circuits in regard to the deviations of their parameters. They are determined by the schematic function properties and are one and the same for all schematic solutions implementing the given function, i.e. they are invariant to the particular circuit implementation. Because of that the mathematical expectation of the schematic function variations is equal to all equivalent transformations in the element bases assumed. However, the sums of the squares of sensitivity functions determining dispersion do not depend on both the schematic function kind and the particular implementation of the frequency selective circuit.

The approach proposed can be applied also to the time characteristics of chains. The obtained dependencies for the relative error with determining the characteristics can be used with the analysis and synthesis of filtrating chains with stable characteristics.

REFERENCES

[1] Kishi G., Kida T. Energy theory of sensitivity in LCR- networks. - IEEE Trans. Circuit Theory, 1987, v. 14, № 12. - p. 380-387

[2] Korn G., T. Korn. Mathematical handbook. McGraw-Hill Book Company, N. Y. 1977

[3] Kida T., Kurogochi K. New sensitivity measures for resistively terminated LC- filters. Int. J. Circuit Theory and Application,1983,v.11,№ 2. - p. 219-234.

[4] Prasad S.C., Singh R.P. Group delay sensitivity- its estimation and application. The Radio and Electronic Engineer, 1981

[5] Wildrick C. Stability of distributed power supply systems, Master's thesis, Virginia Polytechnic Institute and State University, Blacksburg, VA, Jan. 1993.